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International Baccalaureate®
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MATHEMATICS
STANDARD LEVEL
PAPER 2

Thursday 5 May 2011 (morning)

1 hour 30 minutes

Candidate session number

| 0 | 0 | | | | | | | | |

Examination code

| 2 | 2 | 1 | 1 | – | 7 | 3 | 0 | 4 |

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided.*

1. *[Maximum mark: 6]*

    The following diagram shows triangle ABC.
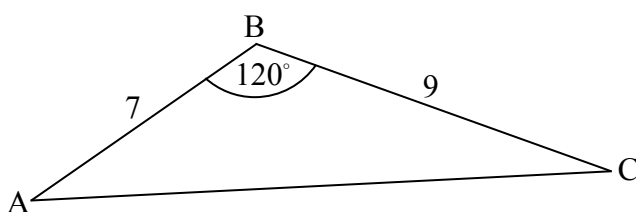
    diagram
    not to scale

    $AB = 7\,cm$, $BC = 9\,cm$ and $A\hat{B}C = 120°$.

    (a) Find AC. *[3 marks]*

    (b) Find $B\hat{A}C$. *[3 marks]*

2. *[Maximum mark: 6]*

Let $f(x) = 3x^2$. The graph of f is translated 1 unit to the right and 2 units down. The graph of g is the image of the graph of f after this translation.

(a)   Write down the coordinates of the vertex of the graph of g.   *[2 marks]*

(b)   Express g in the form $g(x) = 3(x - p)^2 + q$.   *[2 marks]*

The graph of h is the reflection of the graph of g in the x-axis.

(c)   Write down the coordinates of the vertex of the graph of h.   *[2 marks]*

Turn over

3. *[Maximum mark: 5]*

In an arithmetic sequence $u_1 = 7$, $u_{20} = 64$ and $u_n = 3709$.

(a) Find the value of the common difference. *[3 marks]*

(b) Find the value of n. *[2 marks]*

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4.   *[Maximum mark: 8]*
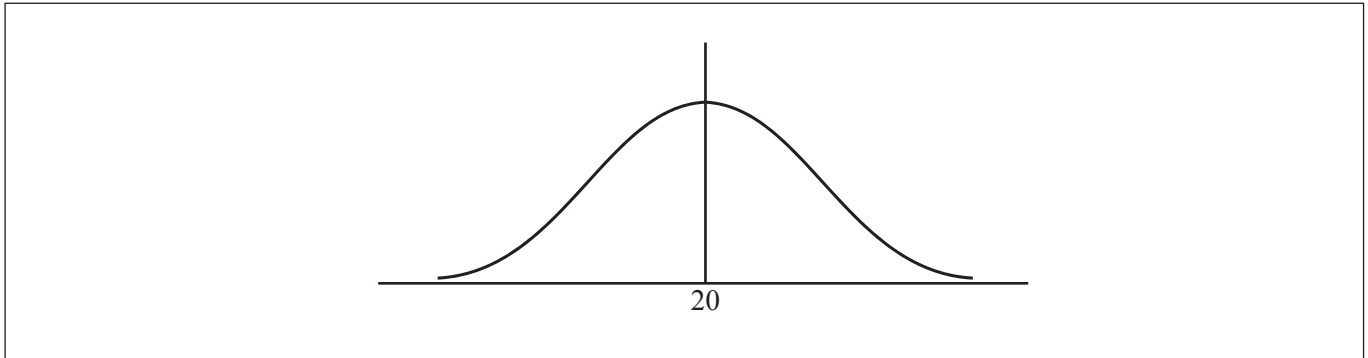
A random variable X is distributed normally with a mean of 20 and variance 9.

(a)   Find $P(X \leq 24.5)$.   *[3 marks]*

(b)   Let $P(X \leq k) = 0.85$.

    (i)   Represent this information on the following diagram.

    (ii)   Find the value of k.   *[5 marks]*

Turn over

5. *[Maximum mark: 7]*

A box holds 240 eggs. The probability that an egg is brown is 0.05.

(a) Find the expected number of brown eggs in the box. *[2 marks]*

(b) Find the probability that there are 15 brown eggs in the box. *[2 marks]*

(c) Find the probability that there are at least 10 brown eggs in the box. *[3 marks]*

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6. *[Maximum mark: 6]*

Let $f(x) = \cos(x^2)$ and $g(x) = e^x$, for $-1.5 \le x \le 0.5$.

Find the area of the region enclosed by the graphs of f and g.

7. *[Maximum mark: 7]*

A company uses two machines, A and B, to make boxes. Machine A makes 60 % of the boxes.

80 % of the boxes made by machine A pass inspection.
90 % of the boxes made by machine B pass inspection.

A box is selected at random.

(a) Find the probability that it passes inspection. *[3 marks]*

(b) The company would like the probability that a box passes inspection to be 0.87. Find the percentage of boxes that should be made by machine B to achieve this. *[4 marks]*

*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

SECTION B

*Answer **all** questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 14]*

The following diagram shows a waterwheel with a bucket. The wheel rotates at a constant rate in an anticlockwise (counterclockwise) direction.

diagram not to scale
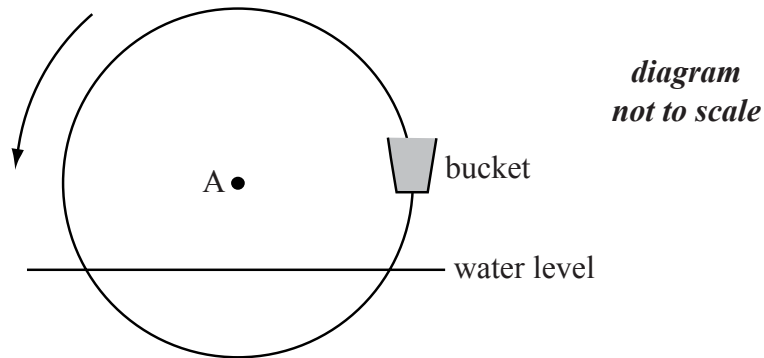
The diameter of the wheel is 8 metres. The centre of the wheel, A, is 2 metres above the water level. After t seconds, the height of the bucket above the water level is given by $h = a \sin bt + 2$.

(a) Show that $a = 4$. *[2 marks]*

The wheel turns at a rate of one rotation every 30 seconds.

(b) Show that $b = \dfrac{\pi}{15}$. *[2 marks]*

In the first rotation, there are two values of t when the bucket is **descending** at a rate of 0.5 ms^{-1}.

(c) Find these values of t. *[6 marks]*

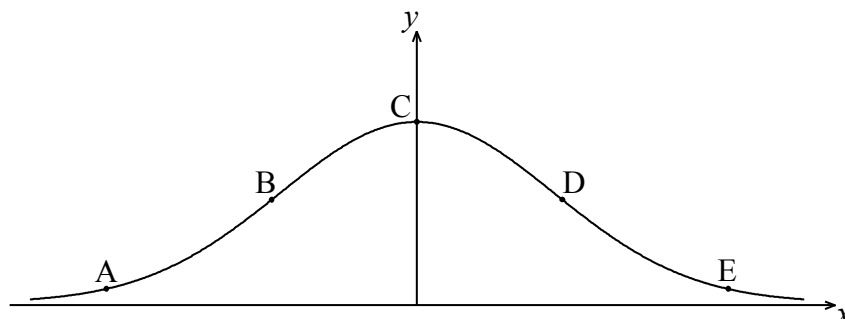(d) Determine whether the bucket is underwater at the second value of t. *[4 marks]*

*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

9.   *[Maximum mark: 15]*

The following diagram shows the graph of $f(x) = e^{-x^2}$.

The points A, B, C, D and E lie on the graph of f. Two of these are points of inflexion.

(a)   Identify the **two** points of inflexion.   *[2 marks]*

(b)   (i)   Find $f'(x)$.

         (ii)   Show that $f''(x) = (4x^2 - 2)e^{-x^2}$.   *[5 marks]*

(c)   Find the x-coordinate of each point of inflexion.   *[4 marks]*

(d)   Use the second derivative to show that one of these points is a point of inflexion.   *[4 marks]*

*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

10. *[Maximum mark: 16]*

Let $f(x) = \log_3 \dfrac{x}{2} + \log_3 16 - \log_3 4$, for $x > 0$.

(a) Show that $f(x) = \log_3 2x$. *[2 marks]*

(b) Find the value of $f(0.5)$ and of $f(4.5)$. *[3 marks]*

The function f can also be written in the form $f(x) = \dfrac{\ln ax}{\ln b}$.

(c) (i)   Write down the value of a and of b.

(ii) Hence on graph paper, **sketch** the graph of f, for $-5 \le x \le 5$, $-5 \le y \le 5$, using a scale of 1 cm to 1 unit on each axis.

(iii) Write down the equation of the asymptote. *[6 marks]*

(d) Write down the value of $f^{-1}(0)$. *[1 mark]*

The point A lies on the graph of f. At A, $x = 4.5$.

(e) On your diagram, sketch the graph of f^{-1}, noting clearly the image of point A. *[4 marks]*

Please **do not** write on this page.

Answers written on this page will
not be marked.